The double intercept form of the equation of a straight line

⇒ In the double intercept form the equation doesn't show the gradient of the line. It shows both the x and the y intercept. i.e, the equation shows where the line cuts the x and y axis.

⇒ The double intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a is the x-intercept and
b is the y-intercept.

⇒ The equation can be written as

$$\left(\frac{1}{a}\right)x + \left(\frac{1}{b}\right)y = 1$$

therefore the intercepts of x and y are actualy the reciprocal of the coefficients of x and y

Summary: If you look keenly, then you'll see that, we are taking the equation in the form $y = mx + c$ and make the left hand side to be 1.

Example:

Find the equation of the line passing through points (5,0) and (0,9) in the double intercept form.

Solution

We need to get the gradient: $\frac{\Delta Y}{\Delta X}$

$$\frac{9-0}{0-5} = -\frac{9}{5}$$, Therefore $m = -\frac{9}{5}$

⇒ The equation: we need a point on the line, a general point and the gradient. Lets take point (0, 9) and (x, y)

Then $$\left(\frac{y-9}{x-0}\right) = -\frac{9}{5}$$

$$\frac{y-9}{\cancel{x}} = -\frac{9}{5}$$ Multiplying both sides by X (the deminator)

$$y - 9 = -\frac{9}{5}x$$, taking -9 on the Right Hand Side.

$$y = -\frac{9}{5}x + 9$$, taking $-\frac{9}{5}x$ to the Left Hand Side

$$y + \frac{9}{5}x = 9:$$ To Make the Right Hand Side to be equal to 1, we divide both sides by 9 or multiply by $\left(\frac{1}{9}\right)$

$$y\left(\frac{1}{9}\right) + \frac{9}{5}\left(\frac{1}{9}\right)x = \cancel{9}\times\left(\frac{1}{\cancel{9}}\right)$$

$$\frac{y}{9} + \frac{x}{5} = 1$$ This is the equation in the double intercept form.

$\frac{y}{9} + \frac{x}{5} = 1$, can be writen as

$\frac{x}{5} + \frac{y}{9} = 1$, where 9 is the y-intercept and 5 is the x-intercept.

Example 2

Find the equation of a line passing through point $\left(\frac{2}{3}, -3\right)$ with a gradient of 2, in the double intercept form.

Solution

$$\frac{y-(-3)}{\left(x-\frac{2}{3}\right)} = 2$$ Simplify the numerator.

$$\frac{y+3}{x-\frac{2}{3}} = 2$$ multiply both sides by the denominator.

$$y+3 = 2\left(x-\frac{2}{3}\right)$$ Open the barxucet

$$y+3 = 2x - \frac{1}{3}$$

$$y = 2x - \frac{1}{3} - 3$$ bring 3 on the Right hand-Size.

$$y = 2x - 3\tfrac{1}{3}$$ Take 2x to the left hand side.

$$y - 2x = -3\tfrac{1}{3}$$

$$y - 2x = -\frac{10}{3}$$ Multiply both sides by $\left(-\frac{3}{10}\right)$.

$$\left(-\frac{3}{10}\right)y - \left(-\frac{3}{10}\right)2x = -\frac{10}{3}\left(-\frac{3}{10}\right)$$

$$-\frac{3}{10}(y) - \left(\frac{-3}{10}\right)2x = -\frac{10}{3}\left(-\frac{3}{10}\right)$$ Simplify the Multiplication

$$\frac{-3y}{10} + \frac{3}{10}(2x) = 1$$ Open the bracket.

$$\frac{-3y}{10} + \frac{6x}{10} = 1$$ Rearrange the work.

$$\frac{6x}{10} - \frac{3y}{10} = 1$$

$$\frac{6}{10}(x) - \frac{3}{10}(y) = 1$$

This is the equation where the x-intercept is $\frac{10}{6}$ and y-intercept is $-\frac{10}{3}$.

Example 3. Rewrite the following equations of straight lines in the double intercept form

(i) $y = 3x + 7$

Solution

$y - 3x = 7$ take 3x to the left

$\frac{y}{7} - \frac{3x}{7} = \frac{7}{7}$ divide each term by 7

$\frac{y}{7} - \frac{3x}{7} = 1$ divide 7 by 7 (to get 1)

$-\frac{3}{7}x + \frac{y}{7} = 1$ Re arrange the work

(ii) $4y + x - 8 = 0$

Solution

$4y + x - 8 = 0$ Take 8 to the Right Hand Sid

$4y + x = 8$ Divide each term by

$\frac{4y}{8} + \frac{x}{8} = \frac{8}{8}$ Divide 8 by 8 to get 1

$\frac{1}{4}y + \frac{x}{8} = 1$

$\frac{y}{4} + \frac{x}{8} = 1$ This is the equation

Parallel and Perpendicular lines:

Parallel lines: Parallel lines are lines whose gradients are equal.

∴ This means that when the equations of parallel lines are written, in the form $y = mx + c$, they have the same value of **m**.

Perpendicular lines: Perpendicular lines are lines that intersect at 90°. **Two** lines are perpendicular to each other if the product of their gradients is -1 i.e

$m_1 \times m_2 = -1$, where m_1 and m_2 are the gradients of the two lines.

Thus in parallel lines $m_1 = m_2$ $\left(\frac{m_1}{m_2} = 1\right)$

in perpendicular lines $m_1 \times m_2 = -1$ $\left(\text{or } m_2 = -\frac{1}{m_1}\right)$

Example: Find the equation of line through (5,2) which is

(a) parallel to $5y - 2x = 10$

(b) Perpendicular to $5y - 2x = 10$

(a) Solution

The line $5y - 2x = 10$, can be written in the form of $y = mx + c$ as

$$5y = 2x + 10$$

$$y = \frac{2x}{5} + \frac{10}{5}$$

$y = \frac{2}{5}x + 2$, From this the gradient is $\frac{2}{5}$

Since the line is parallel to $5y - 2x = 10$, its gradient is $\frac{2}{5}$, and it passes through (5, 2). The equation is

$$\frac{y-2}{x-5} = \frac{2}{5}$$

$$y - 2 = \frac{2}{5}(x-5)$$

$$y - 2 = \frac{2}{5}x - 2$$

$$y = \frac{2}{5}x - 2 + 2$$

$$y = \frac{2}{5}x$$

(b) Solution

Again the line $5y - 2x = 10$ can be written in the form $y = mx + c$ as

$$y = \frac{2x}{5} + 2$$ from this the gradient is $2/5$.

Since the line is parallel to the given line, the product of their gradients is -1

i.e $m_1 \times m_2 = -1$

Thus if $m_1 = 2/5$, we can find m_2

$$2/5 \times m_2 = -1$$
$$(5/2)\, 2/5 \times m_2 = -1 \times (5/2)$$
$$m_2 = -5/2$$

or

$$m_2 = -\frac{1}{m_1}$$
$$m_2 = \left(\frac{-1}{2/5}\right)$$
$$m_2 = -1 \times 5/2$$
$$= -5/2$$

⇒ Therefore we can now find the equation of the line passing through $(5, 2)$ with the gradient $-5/2$

$$\frac{y-2}{x-5} = -5/2$$
$$y - 2 = -5/2\,(x - 5)$$
$$y - 2 = -5/2\,x + 25/2$$
$$y = -5/2\,x + 12.5 + 2$$
$$y = -5/2\,x + 14.2 \Rightarrow y = -\frac{5}{2}x + \frac{29}{2}$$

Assignment:

From Adv Bk 2 Mathematics

⇒ Exercise 5.4 page 41

No 1. a, c

3. a, c, d

4. a, c

5. c, d

⇒ Exercise 5.5 page 43.

No 1, a, b

2. a, b

3. c, d

Or

For those with K.L.B Bk 2

Exercise 4.3.

No: 3, a b c d

Exercise 4.4.

1. a, g

2, c e

Exercise 4.5

1. a, d,

2. c, f